# Probing the quark mass in elastic and transition form factors

Ian Cloët

(University of Washington)

Collaborators

Craig Roberts (ANL)

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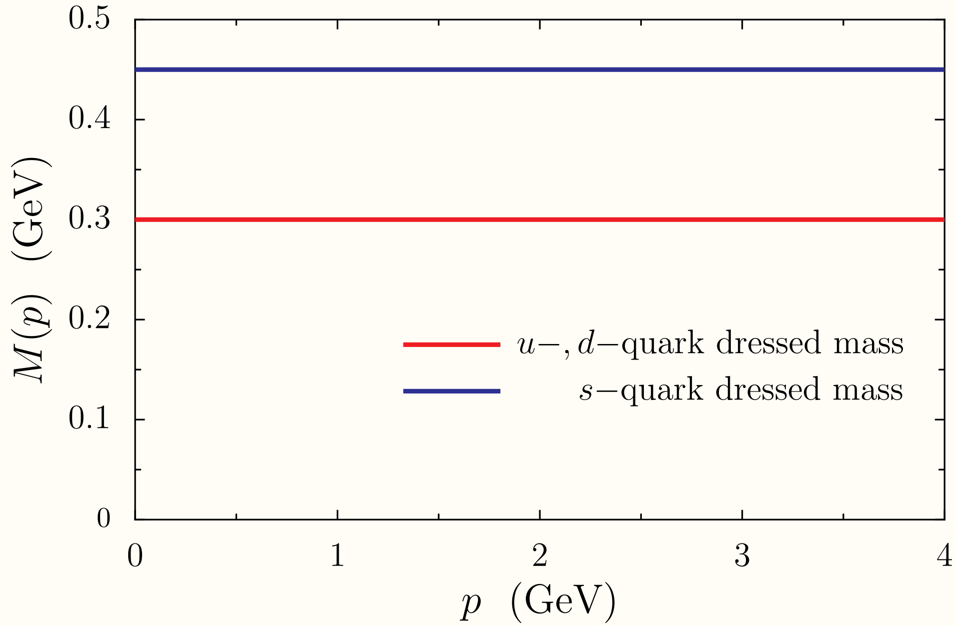
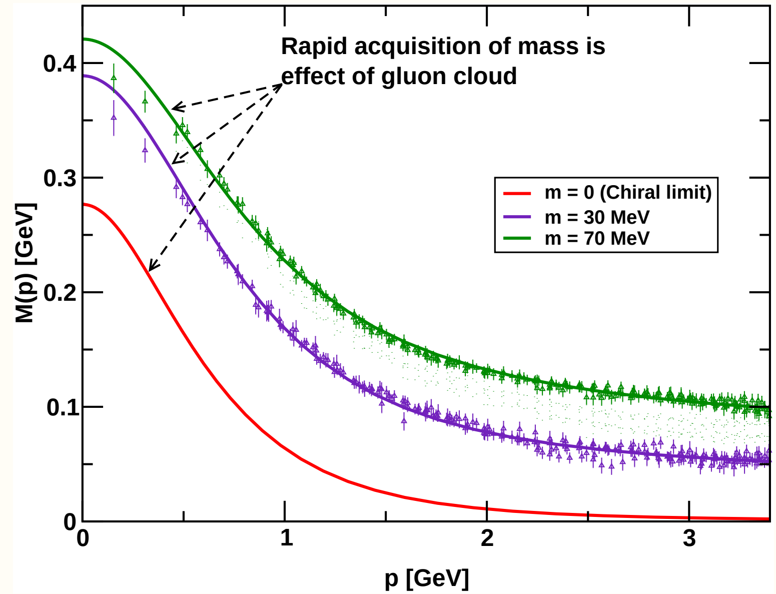
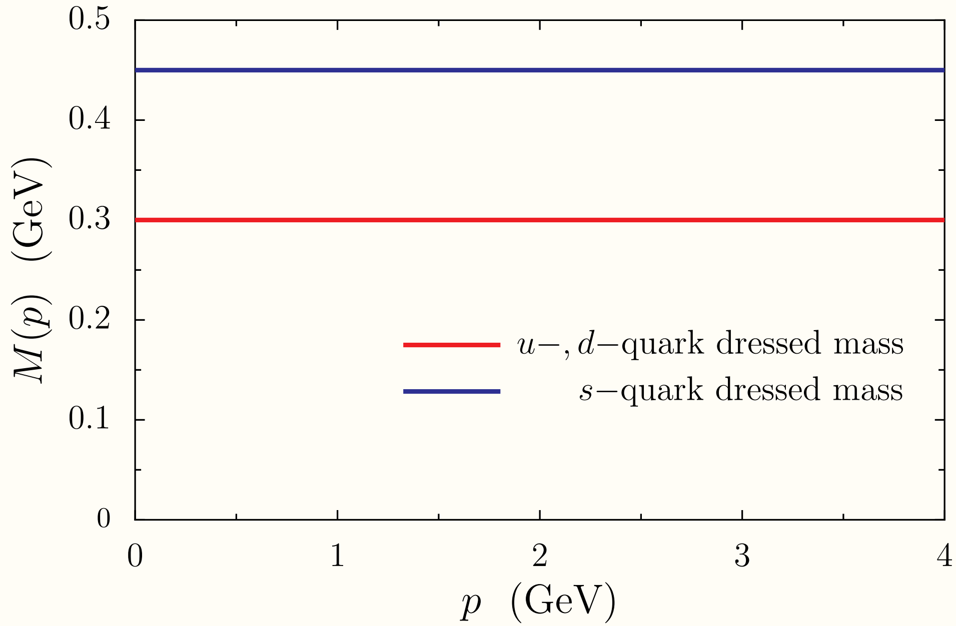
# The Quark Mass Function

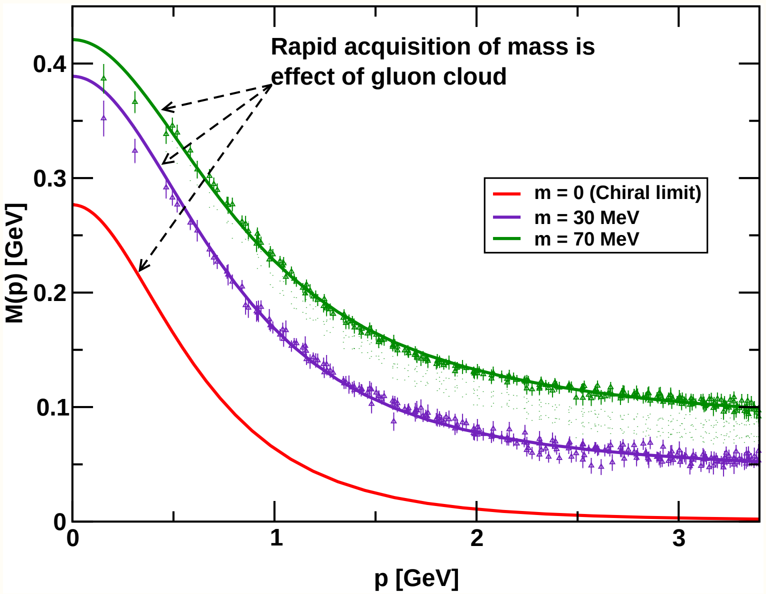


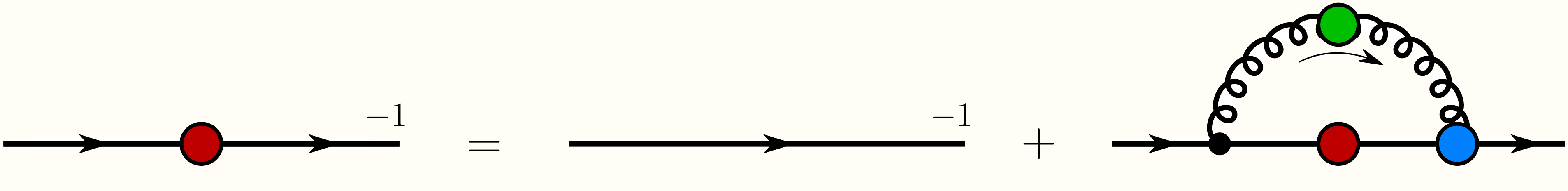

- Quark propagator is fundamental when building models of QCD
  - In QCD $M(p^2)$ must run & give perturbative limit
- A constant constituent mass is phenomenologically successful
  - Constituent quark models for spectroscopy
  - NJL models for meson and baryon static properties
- Can we identify observables that are sensitive to $M(p^2)$
  - If so can experiment help constrain $M(p^2)$ within DSE framework

# *Roadmap*



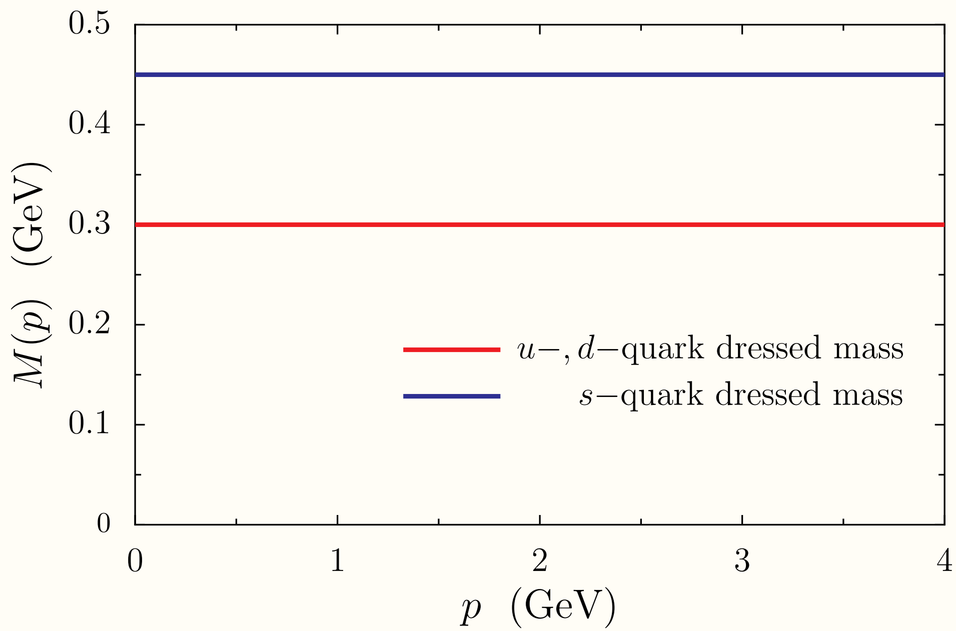



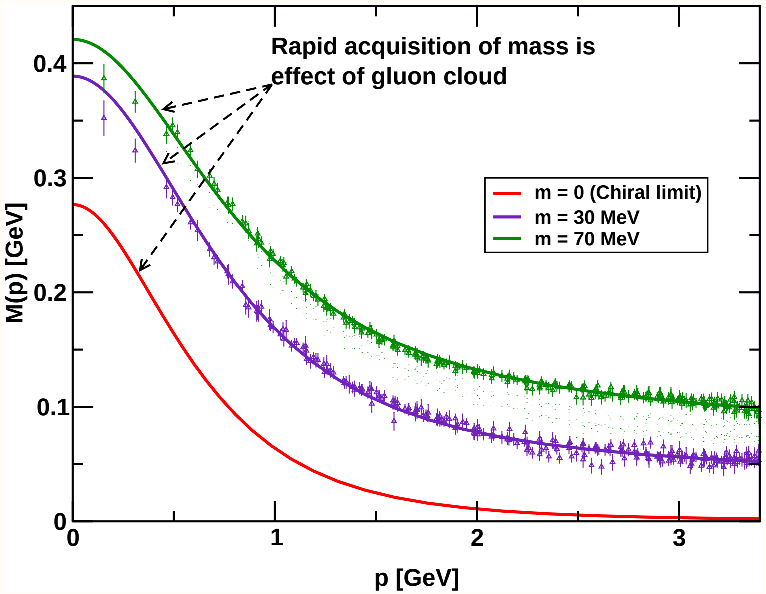

$$\frac{1}{m_G^2}\, g_{\mu\nu}\, \gamma^{\nu} \qquad\qquad g^2\, D_{\mu\nu}(p-k)\Gamma^{\nu}(p,k)$$




- QCDs Gap Equation: $S(p) = Z(p^2)/\left[i\not{p} + M(p^2)\right]$

# Roadmap

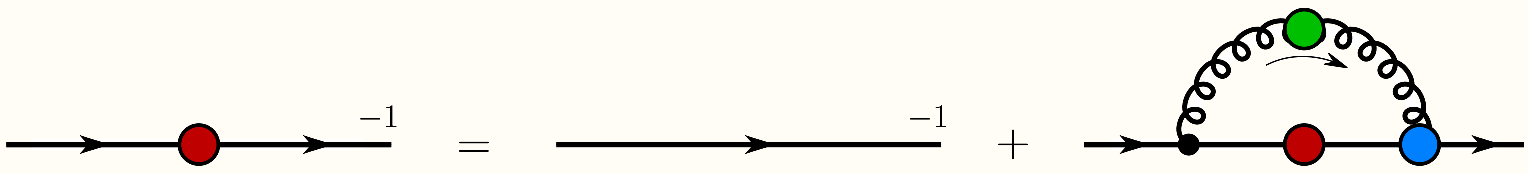



$$\frac{1}{m_G^2}\, g_{\mu\nu}\, \gamma^\nu \qquad\qquad g^2\, D_{\mu\nu}(p-k)\Gamma^\nu(p,k)$$

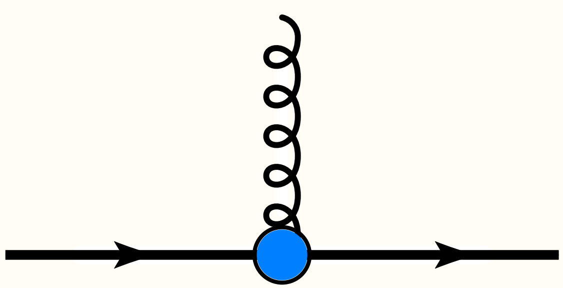
- Compare observables within one framework with different interactions
- Experiment will constrain interaction $\Longleftrightarrow$ quark–gluon vertex
- Knowledge of quark–gluon vertex provides $\alpha_s(Q^2)$ within DSEs
  - also gives $\beta$-function and may shed light on confinement

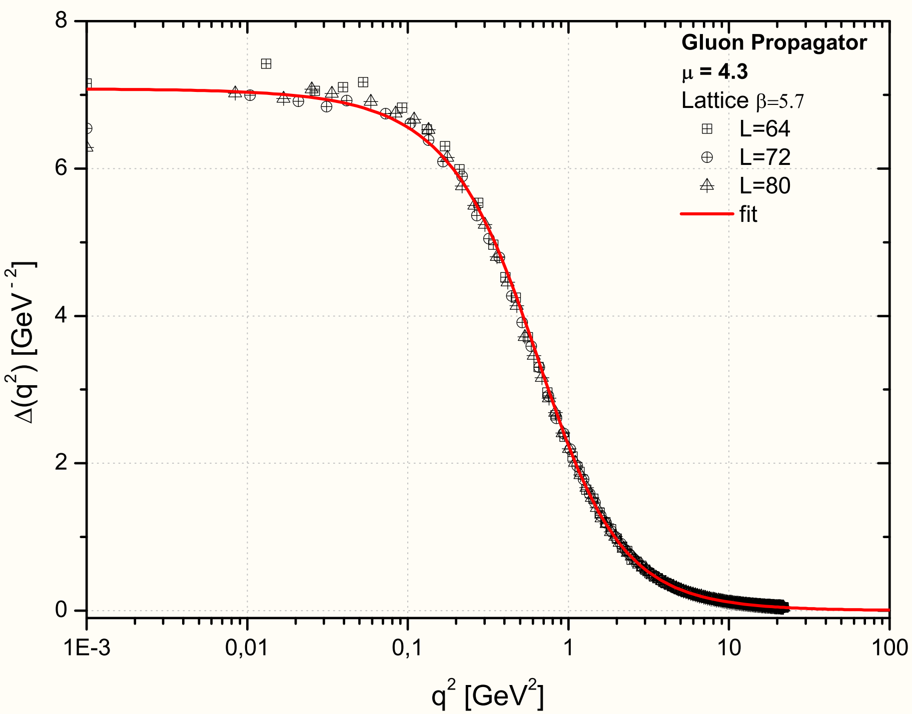
# Quark DSE $\Longleftrightarrow$ Gap Equation




$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)} \qquad\qquad D(p) = \left(\delta^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}\right)\Delta(p^2)$$

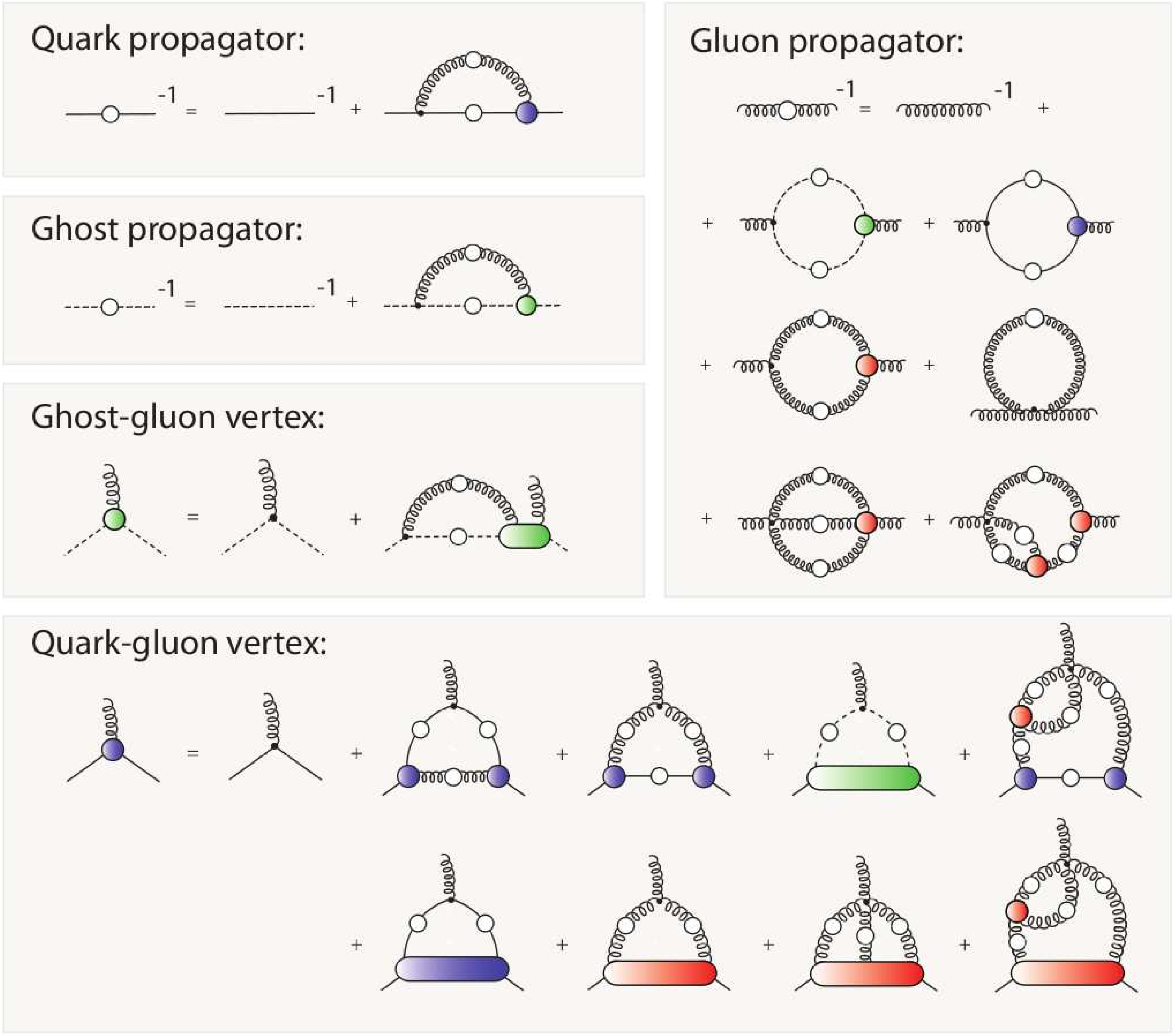
- Given a quark–gluon vertex we can solve for $M(p^2)$






A. C. Aguilar *et al*, Phys. Rev. **D81**, 034003 (2010).

# Quark DSE $\Longleftrightarrow$ Gap Equation



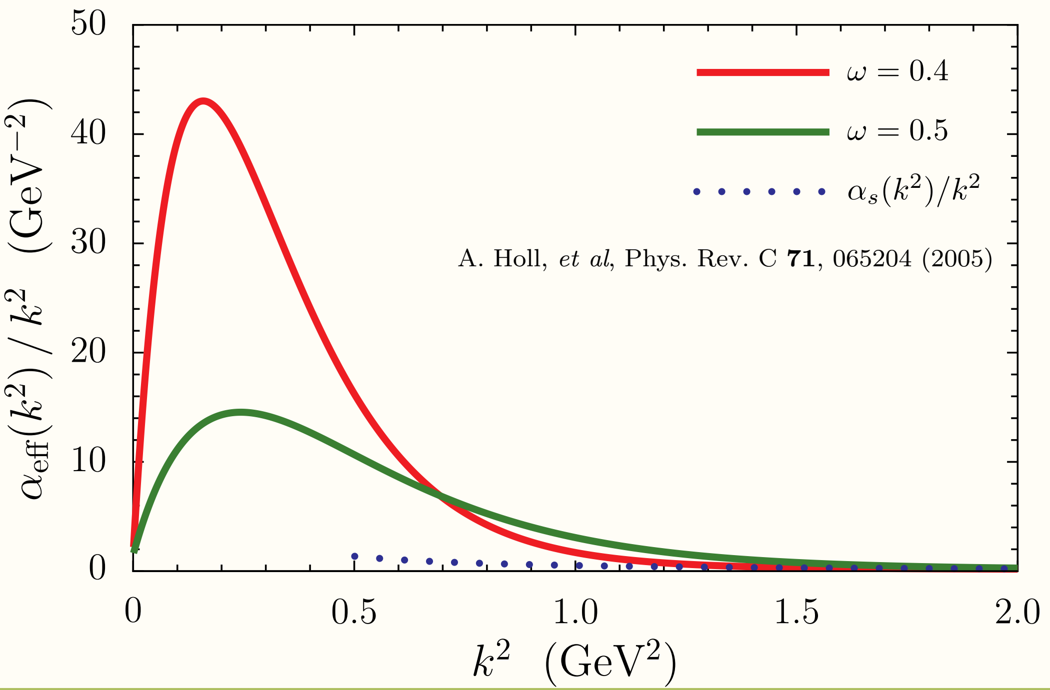
# DSE and the Maris–Tandy Model

- Clearly need a sensible truncation scheme
  - must maintain symmetries of theory
  - rainbow-ladder truncation is one such scheme
- Maris–Tandy – ansätze for gluon propagator and quark-gluon vertex

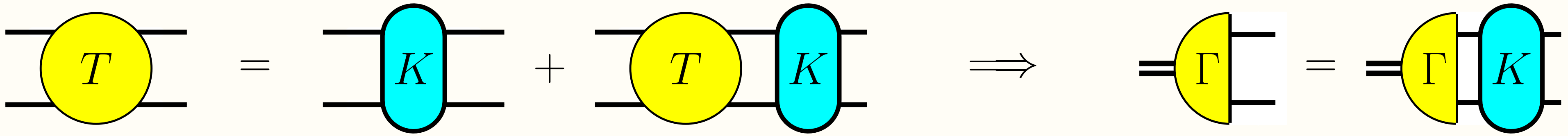
$$\frac{1}{4\pi} g^2 D_{\mu\nu}(p-k)\,\Gamma_\nu(p,k) \quad \longrightarrow \quad \alpha_{\text{eff}}(p-k)\, D^{\text{free}}_{\mu\nu}(p-k)\,\gamma_\nu$$

- Build in the correct perturbative limit

$$\alpha_{\text{eff}}(k^2) \overset{k^2\to\infty}{\longrightarrow} \frac{\pi\,\gamma_m}{\ln\left(k^2/\Lambda^2_{\text{QCD}}\right)}$$



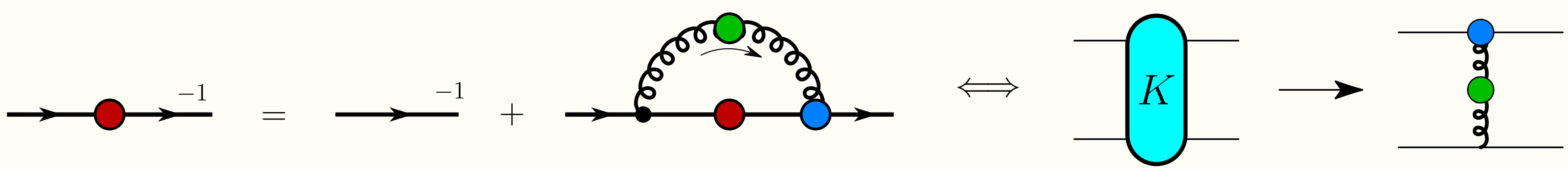
# Mesons and the Bethe-Salpeter Equation




- Mesons show up as poles in the two-body $T$-matrix
- What is the BSE kernel: must preserve symmetries
  - e.g. Axial–Vector Ward–Takahashi Identity

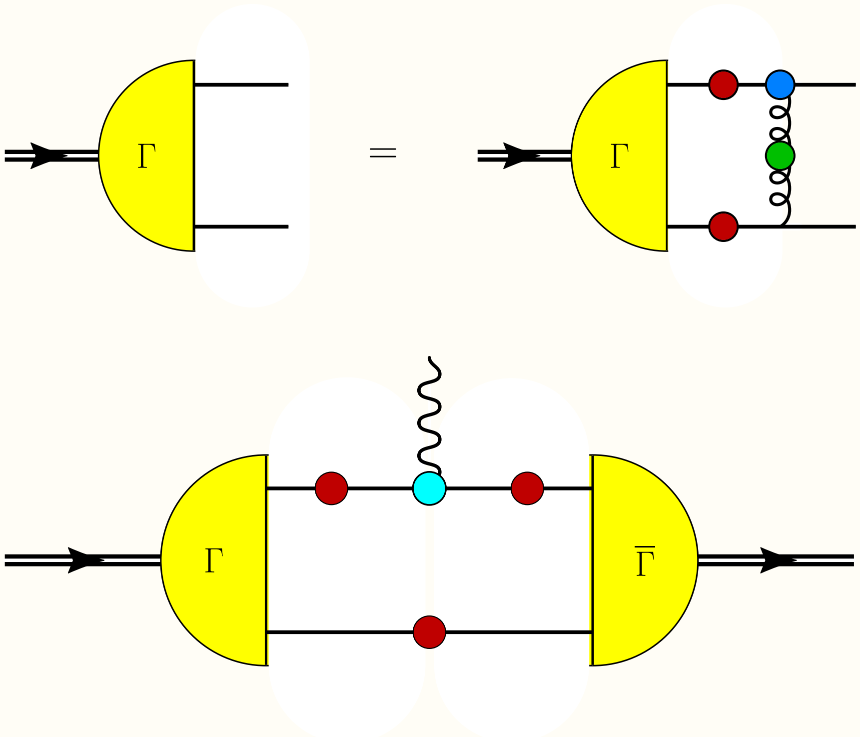
$$q_\mu \, \Gamma_5^{\mu,i}(p',p) = S^{-1}(p')\,\gamma_5\,\tfrac{1}{2}\tau_i + \tfrac{1}{2}\tau_i\,\gamma_5\,S^{-1}(p) + 2\,m\,\Gamma_\pi^i(p',p)$$

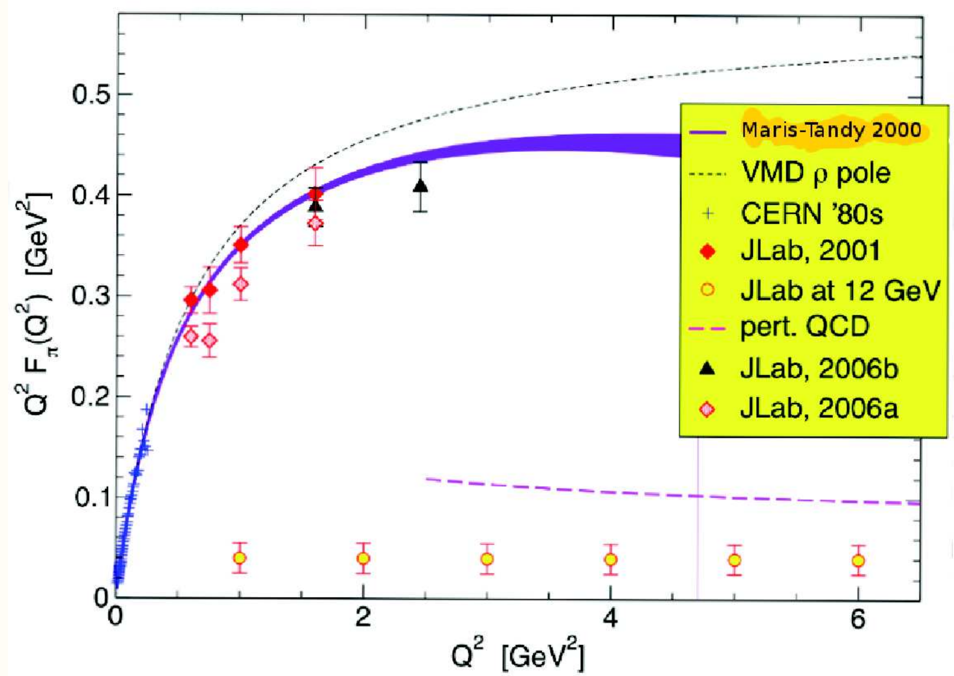
- Kernels of gap and BSE must be intimately related




- Maris–Tandy: excellent description of light pseudoscalar and vector mesons – 31 masses/couplings/radii with rms error of 15%

# *Pion form factor*



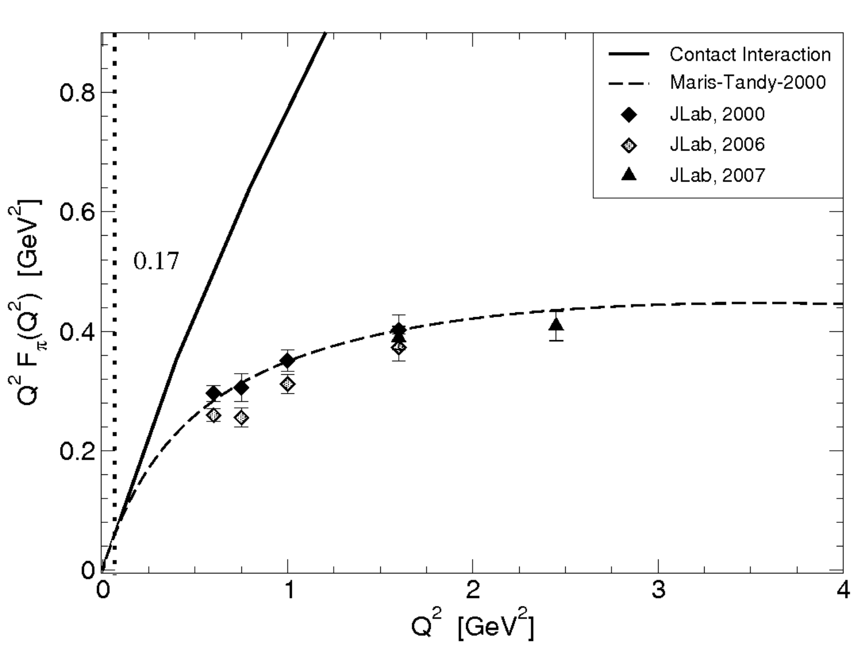

- Pion BSE vertex has the general form

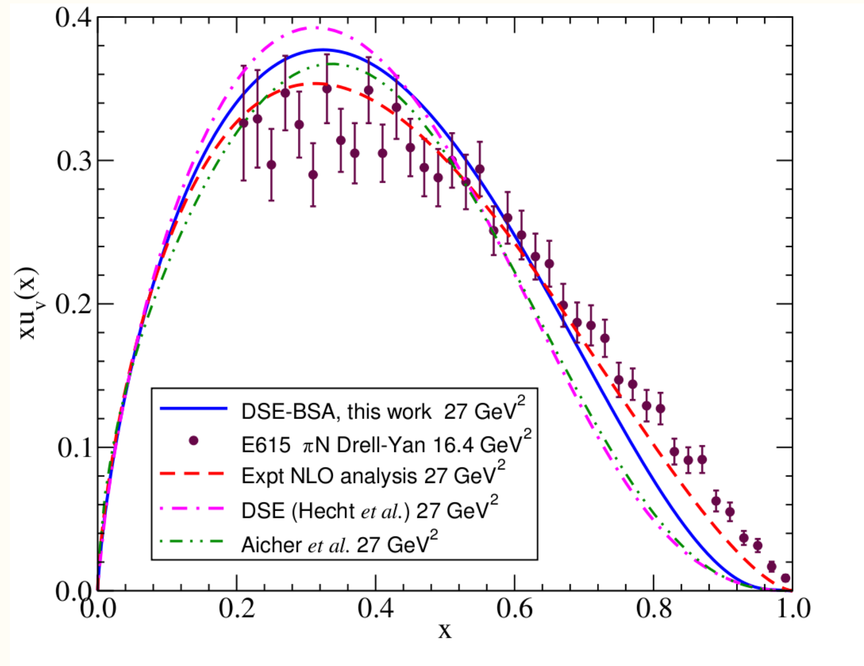
$$\Gamma_\pi(p,k) = \gamma_5 \Big[ E_\pi(p,k) + \not{p}\, F_\pi(p,k) + \not{k}\, k\cdot p\, \mathcal{G}(p,k) + \sigma^{\mu\nu} k_\mu p_\nu\, \mathcal{H}(p,k) \Big]$$

- Use Ball-Chiu Ansatz for quark–photon vertex: satisfies WTI

$$\Gamma^\mu_{\text{BC}}(p',p) = \gamma^\mu\, \Sigma_A(p'^2,p^2) + P^\mu\, \Delta_B(p'^2,p^2) + P^\mu \not{P}\, \Delta_A(p'^2,p^2)$$

# Some Consequences of Running Quark Mass




- L. X. Gutierrez-Guerrero *el al.*, Phys. Rev. **C81**, 065202 (2010) [arXiv:1002.1968 [nucl-th]].
- T. Nguyen, A. Bashir, C. D. Roberts, P. C. Tandy,[arXiv:1102.2448 [nucl-th]]
- In gap equation use simpler kernel

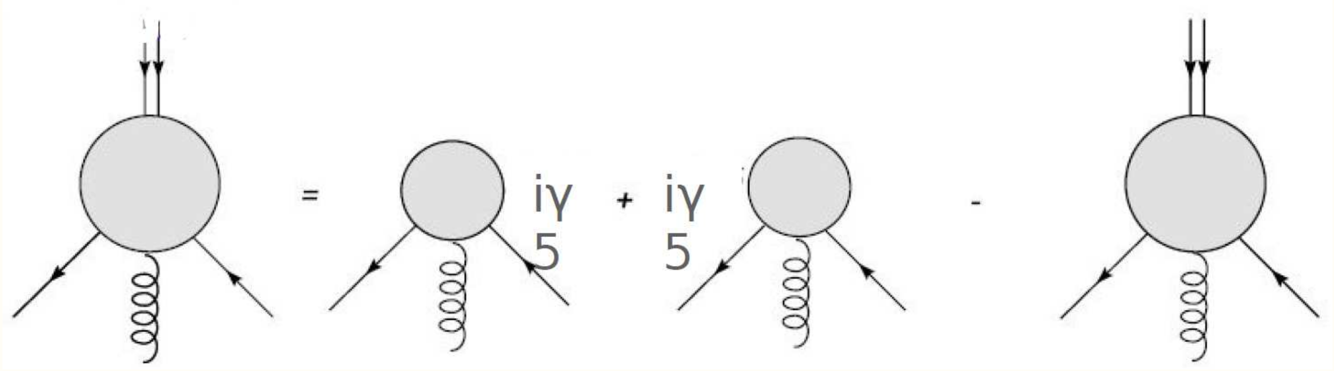
$$g^2\, D_{\mu\nu}(p-k)\Gamma^\nu(p,k) \;\to\; \frac{1}{m_G^2}\, g_{\mu\nu}\, \gamma^\nu$$

  - Quark no longer has a running mass

- Pion PDF $x \to 1$: contact – $q(x) \sim (1-x)^1$; DSE – $q(x) \sim (1-x)^2$

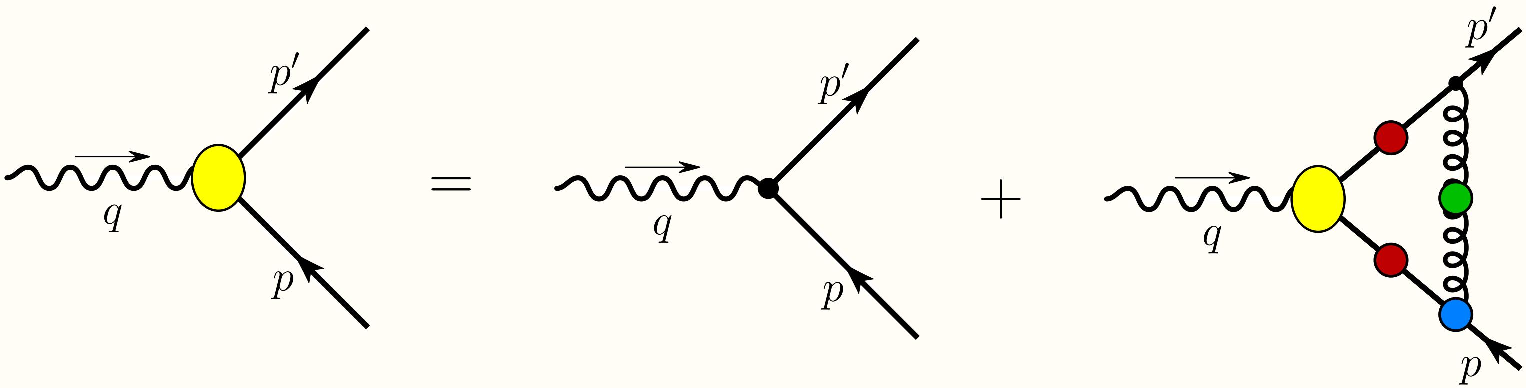
# *Toward a General Quark–Gluon Vertex*

- Maris–Tandy has been successful, however it does breakdown
  - e.g. excited states, $\rho - a_1$ mass splitting, ...
- Clear signal that the Maris–Tandy quark–gluon vertex is too simple
- Inability to construct new Bethe–Salpeter kernel blocked progress
- However, it is now possible to formulate an Ansatz for Bethe-Salpeter kernel given any form for the dressed-quark-gluon vertex




  - L. Chang and C. D. Roberts, Phys. Rev. Lett. **103**, 081601 (2009)
- This enables direct connection between experiment and a general quark–gluon vertex with DSE framework

# Quark–Gluon and Quark–Photon Vertices




- Quark–gluon and quark–photon vertices have same Lorentz structure

$$\Gamma^\mu(p',p) = \sum_{i=1}^{12} \lambda_i^\mu \, f_i(p'^2,p^2,q^2) = \Gamma_L^\mu + \Gamma_T^\mu$$

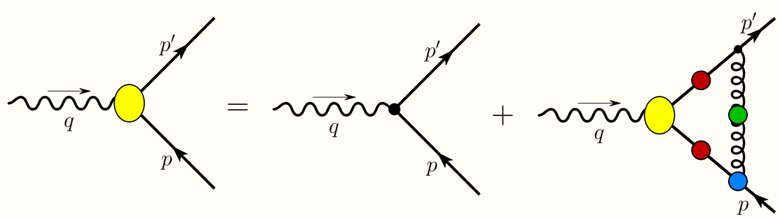
- Coupling of photon to quark is given by inhomogeneous BSE
  - properties dictated by quark propagator and quark–gluon vertex
- Ward-Takahashi identity constrains $\Gamma_L^\mu$ for quark–photon vertex

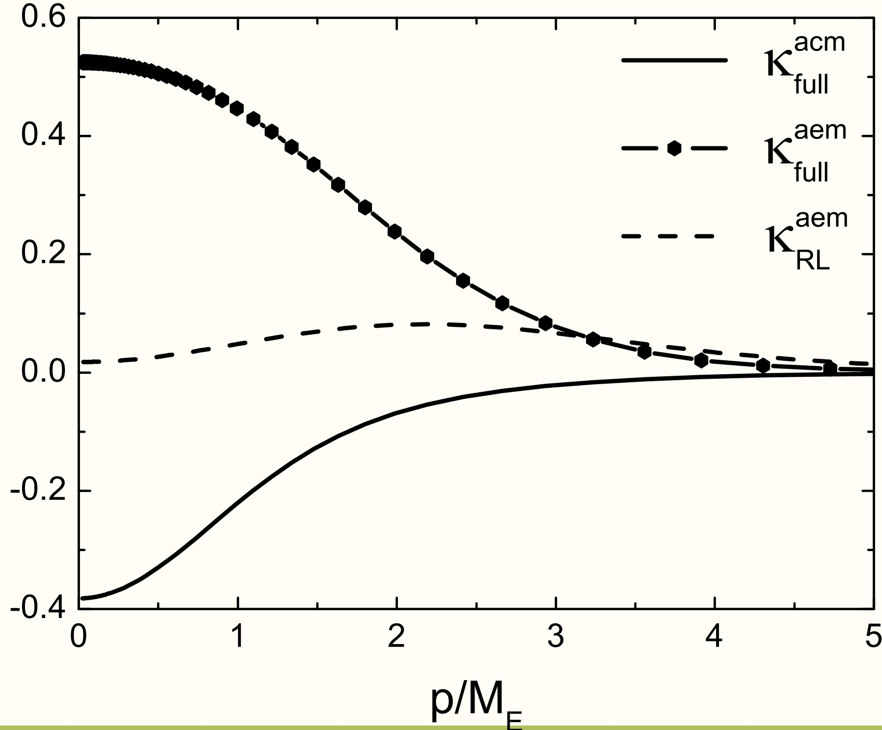
$$q_\mu \, \Gamma^\mu_{\gamma qq} = q_\mu \, \Gamma^\mu_L = \hat{Q} \, \left[ S^{-1}(p') - S^{-1}(p) \right] , \qquad q_\mu \, \Gamma^\mu_T = 0$$

- Constituent quarks are strongly dressed by gluons
  - therefore expect sizable transverse form factors – c.f. nucleon

# Dressed Quark Anomalous Magnetic Moment

- Include $\sigma^{\mu\nu} q_\nu \tau_5(p', p)$ [anomalous chromomagnetic] term in quark–gluon vertex
  - has been absent from previous calculations
- Generates anomalous electromagnetic term in quark–photon vertex
- Confined quarks $\Longrightarrow$ no mass shell – anomalous mm ill defined
  - however associate with $i\sigma^{\mu\nu} q_\nu$ piece of quark–photon vertex

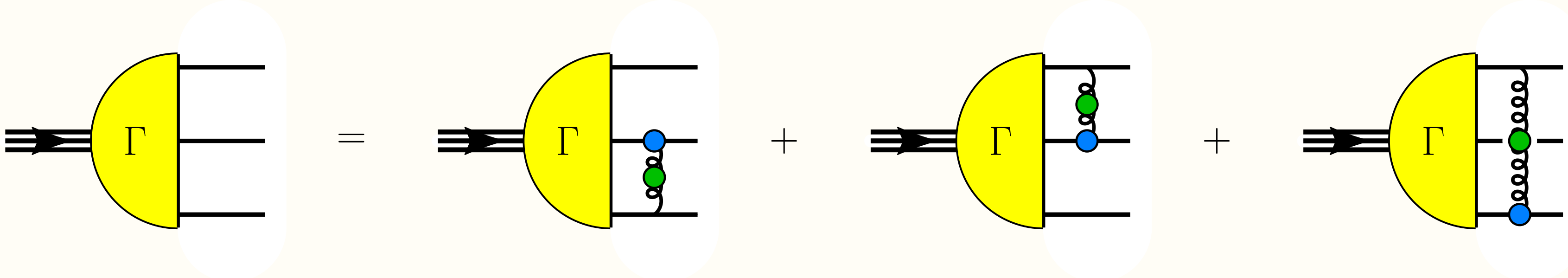



- L. Chang, Y. -X. Liu, C. D. Roberts, Phys. Rev. Lett. **106**, 072001 (2011).
- Investigate effect on nucleon form factors

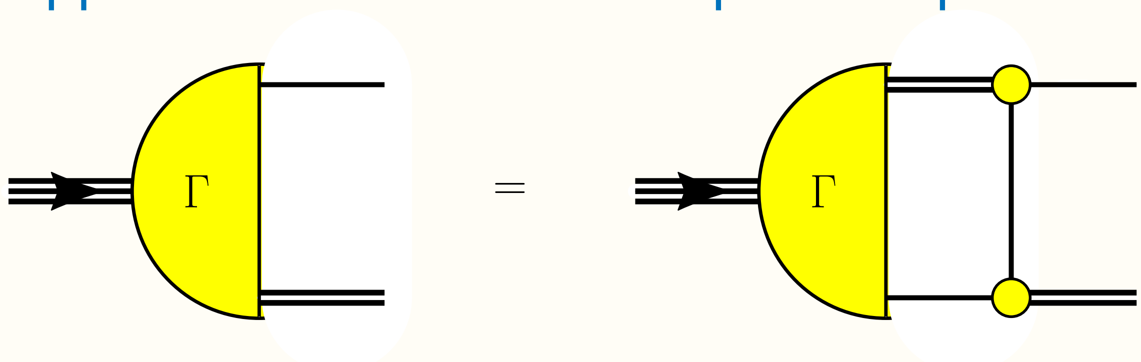
# *Nucleon and the Faddeev Equation*

- Consistency $\Longrightarrow$ solve Faddeev Equation with DSE kernel




  - G. Eichmann *et al.*, Phys. Rev. Lett. **104**, 201601 (2010).

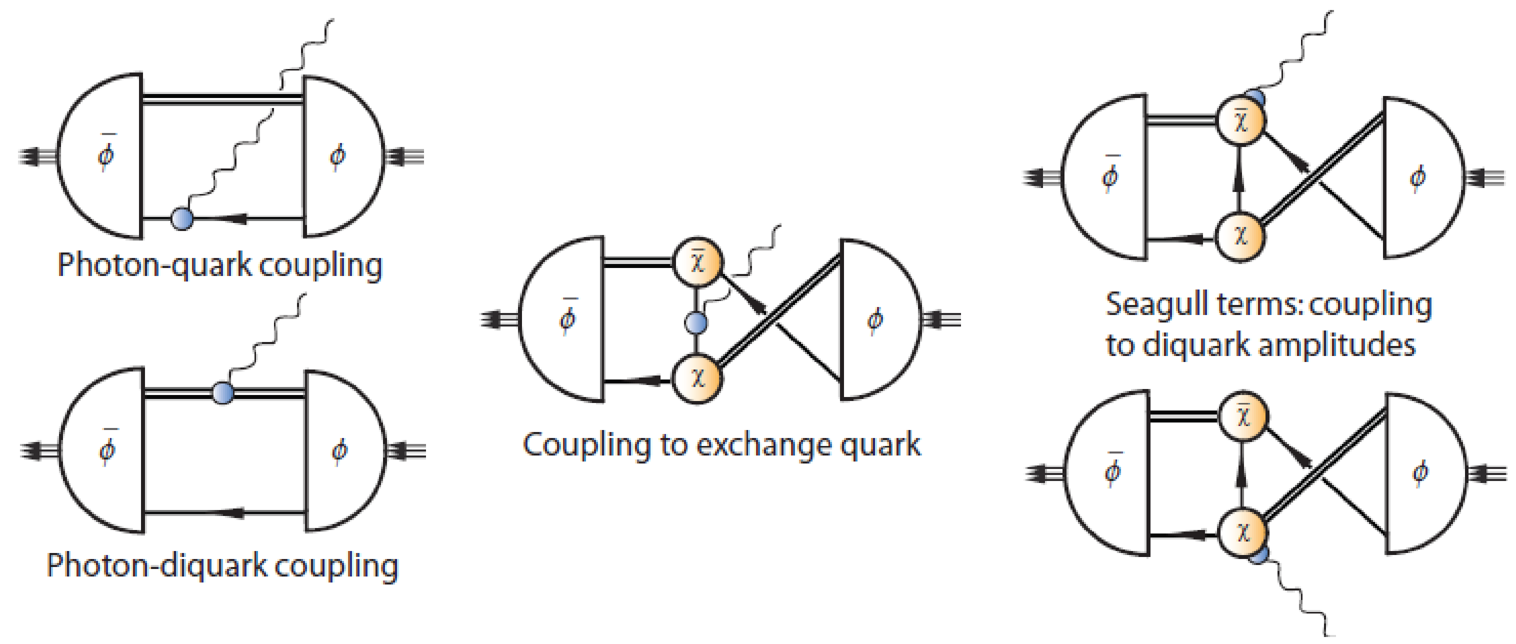
- Instead we approximate nucleon as a quark–diquark bound state




- Include scalar and axial-vector diquarks
- For masses quark–diquark approx results agree to within 5%
- Equation has discrete solutions at $p^2 = m_i^2$; nucleon, roper, etc
  - Yields Faddeev amplitude describes quark-diquark relative motion

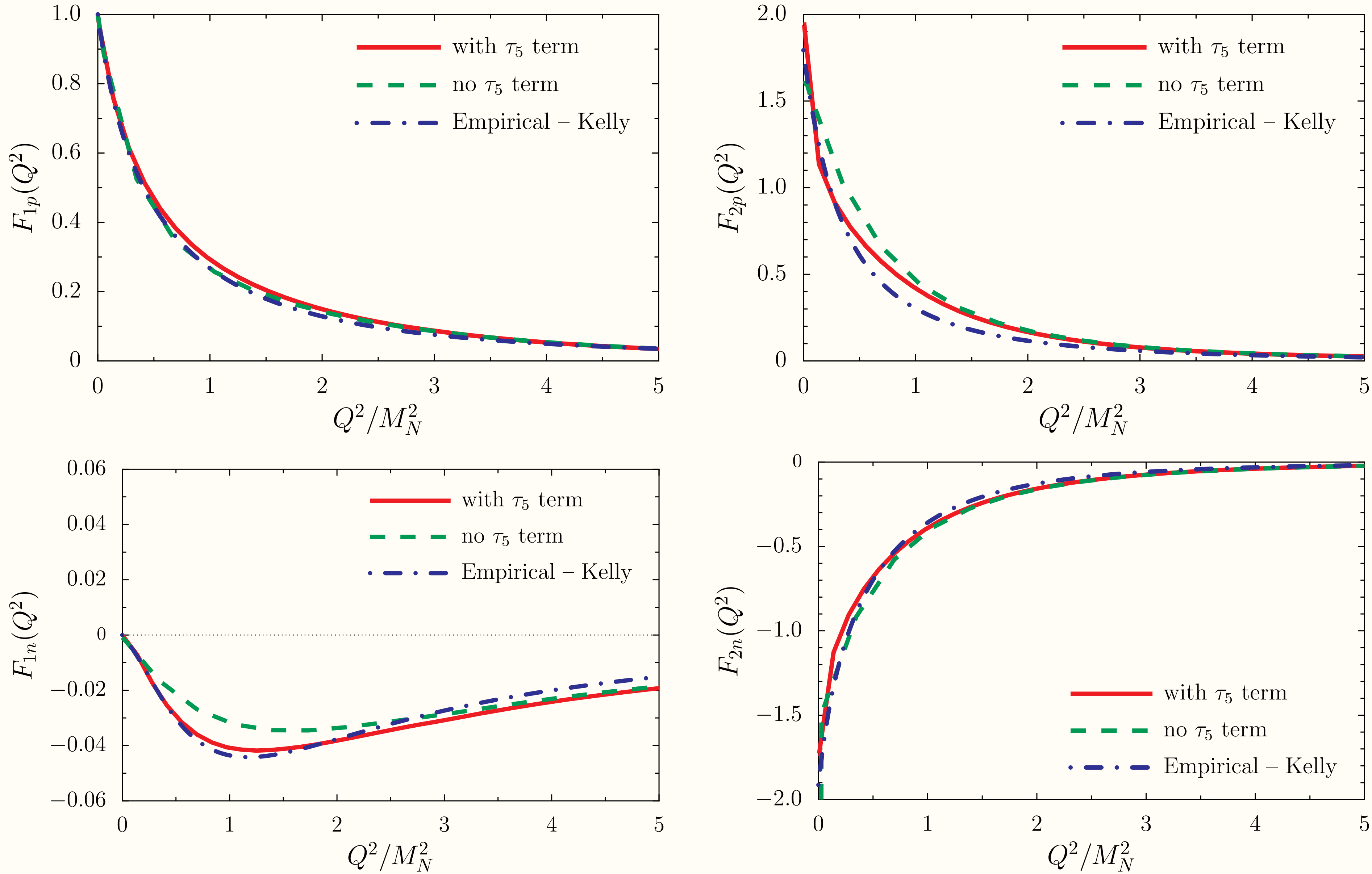
# *Nucleon Electromagnetic Current*

- Current conservation requires the following diagrams:



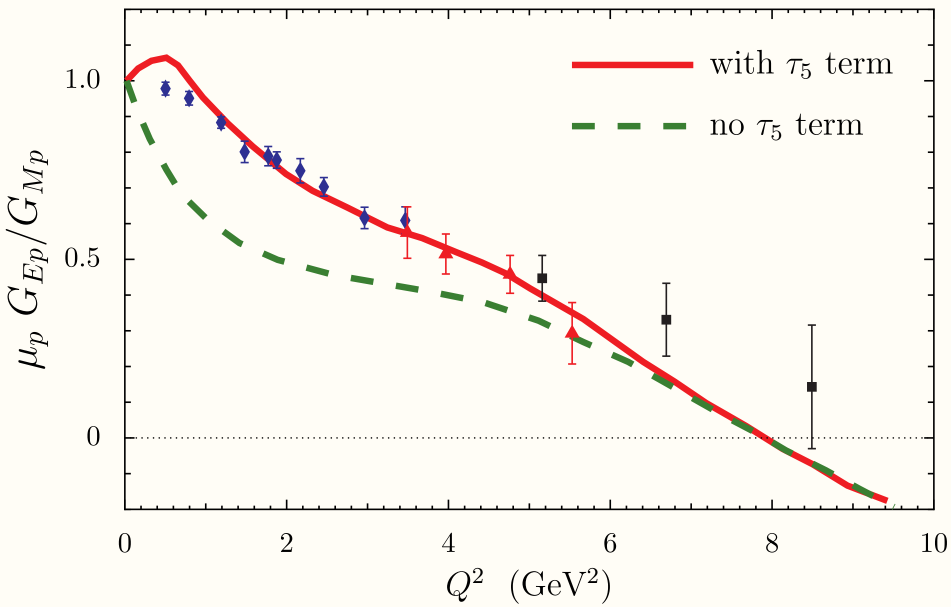

- Dressed quark–photon vertex
  - longitudinal piece, $\Gamma_L^\mu$, constrained by WTI
  - transverse piece, $\Gamma_T^\mu$, include $i\sigma^{\mu\nu}q_\nu$ term
- Predictions for nucleon form factors to $Q^2 \sim 10 - 15$ GeV$^2$

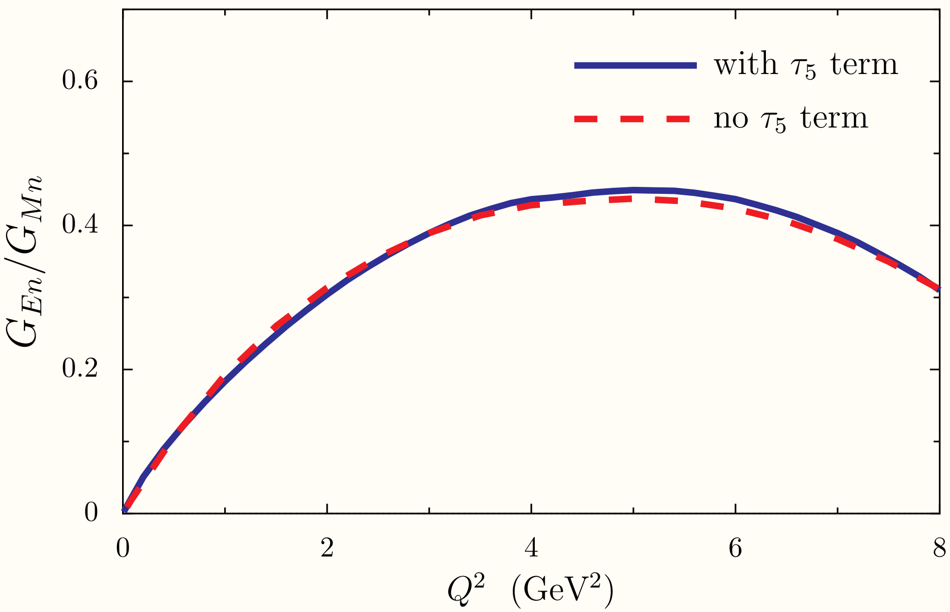
# Nucleon Form Factors Results




- $\tau_5$ is the anomalous magnetic moment term in quark–photon vertex

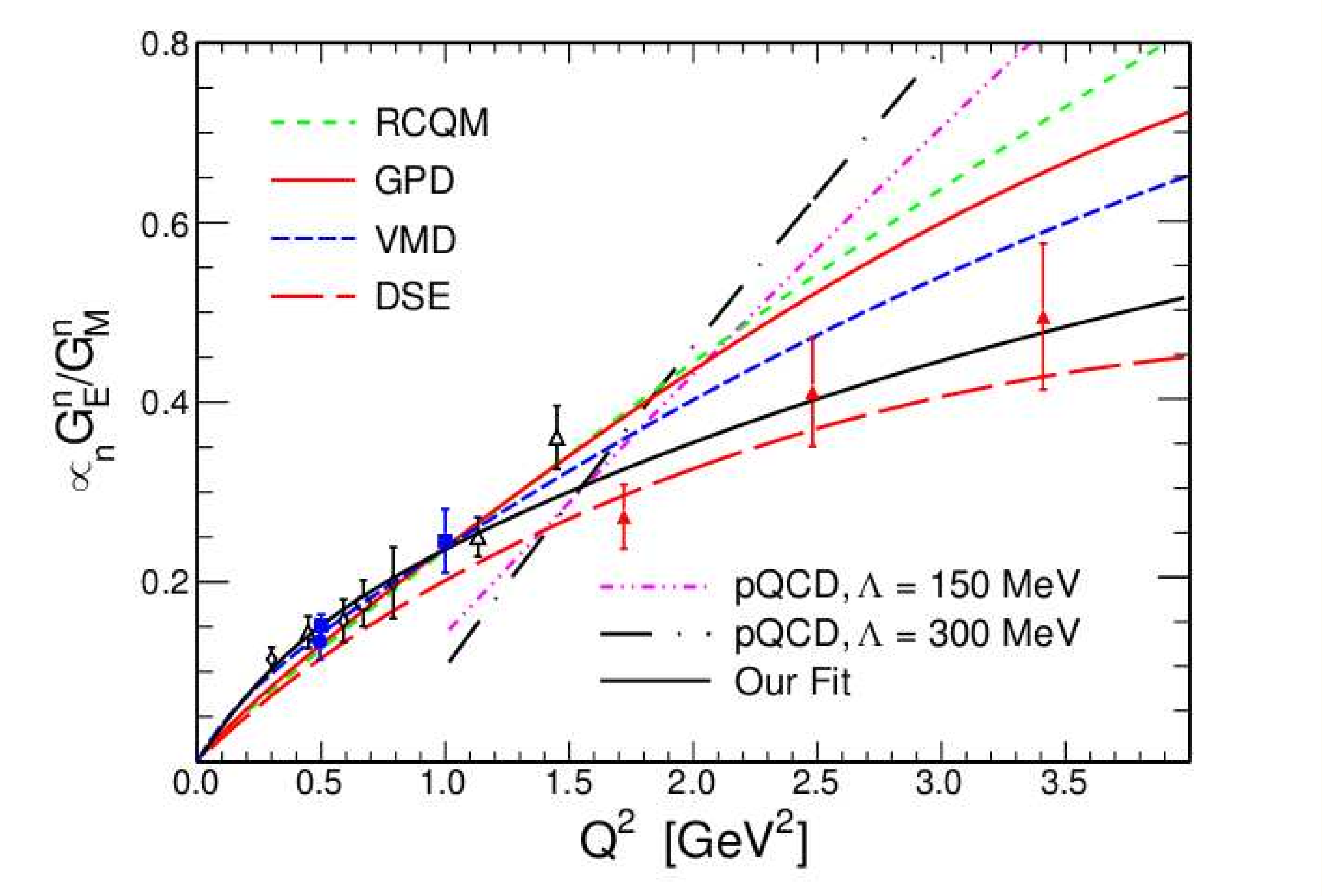
# Nucleon Form Factors



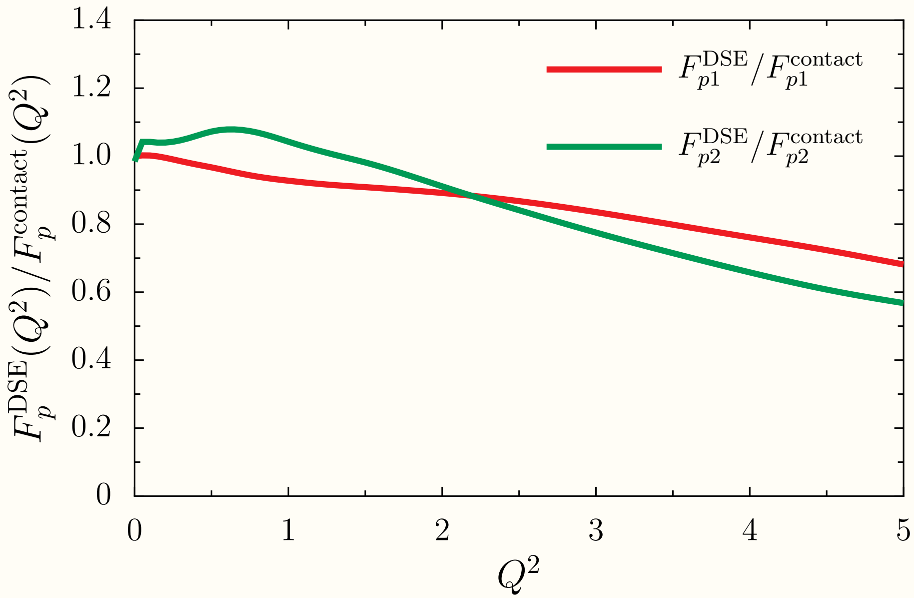

- DSE results now include the anomalous electromagnetic term
  - important for low to moderate $Q^2$
- Reasonable description of nucleon form factors
- DSE model for nucleon can be improved
  - need to include $\rho$ and $\omega$ contribution to $\Gamma^{\mu}_{\gamma qq}$

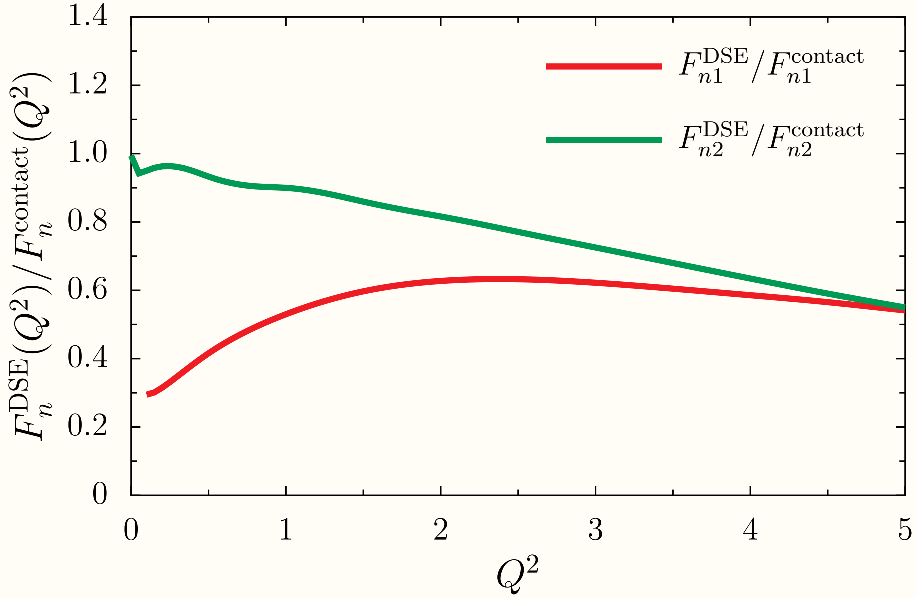
# *Nucleon Form Factors, cont'd*




- S. Riordan, *et al* Phys. Rev. Lett. **105**, 262302 (2010)
- DSE prediction agrees with this recent data

# Comparison with Constant Mass Function




- Find that at $Q^2 = 0$ two results agree rather well
- Reinforces the notion that a constant constituent mass is a reasonable approximation to low energy QCD
  - provided symmetries are preserved
  - good for calculating static properties: mag. moments, PDFs, etc
- However for $Q^2 \neq 0$ operators – running mass is important

# The $N^*$ (Roper) Resonance

- $N^*$ manifests as second pole in Faddeev equation kernel
  - $M_N = 0.940$ GeV and $M_{N^*} = 1.8$ GeV
  - Agrees very well with EBAC value for quark core mass
- "Wavefunction" is given by eigenvector at pole: $p^2 = m_i^2$
- For contact model $N$, $N^*$ "wavefunction" has the simple form

$$\Gamma(p) = \begin{bmatrix} \alpha_1 \\ \alpha_2 \frac{p^\mu}{M} \gamma_5 + \alpha_3 \gamma^\mu \gamma_5 \end{bmatrix} u(p)$$

- For the nucleon: $\alpha_1 = 0.43, \quad \alpha_2 = 0.024, \quad \alpha_3 = -0.45$
- For the Roper: $\alpha_1 = 0.0011, \quad \alpha_2 = 0.94, \quad \alpha_3 = -0.051$
- For nucleon scalar and axial–vector diquarks equally dominant
- However, $N^*$ is complete dominated by the axial–vector diquark

# *A Radial Excitation*

- Nucleon and Roper angular momentum must satisfy:

$$J = \frac{1}{2} = \frac{1}{2}\,\Delta\Sigma + L_q + J_g$$

- For nucleon experiment gives

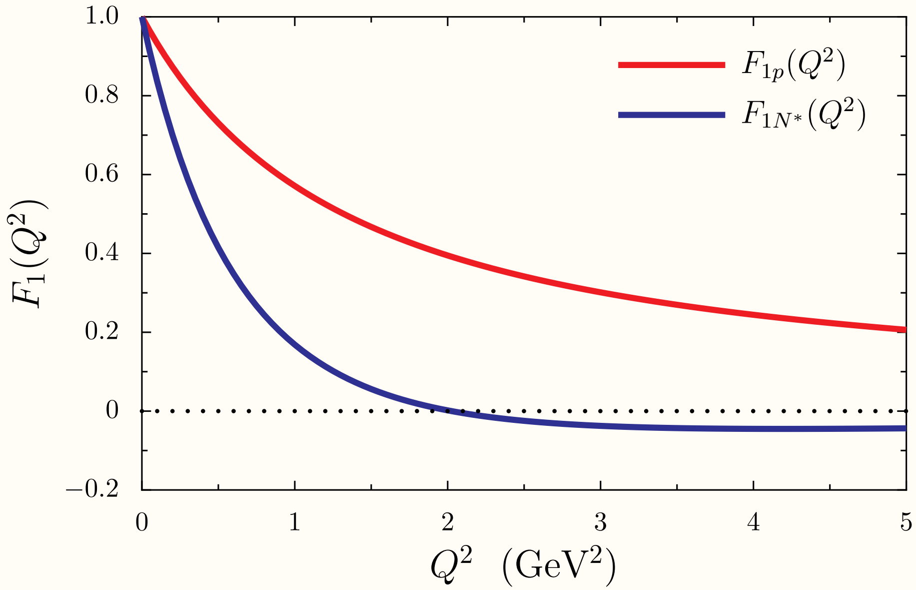
$$\Delta\Sigma = 0.33 \pm 0.03(stat.) \pm 0.05(syst.)$$ [COMPASS & HERMES]

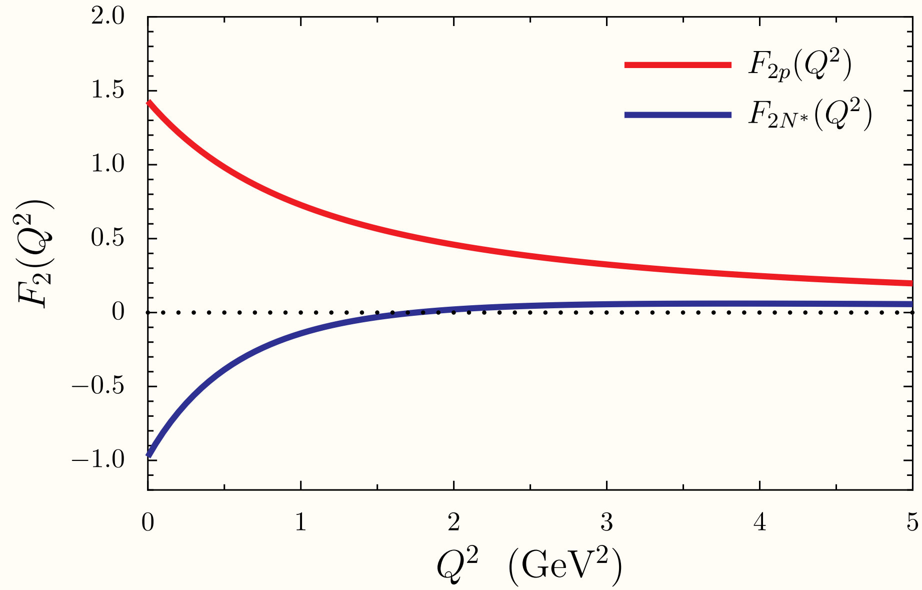
- Contact interaction gives:

$$\Delta\Sigma_N = 0.68 - 0.21 = 0.47, \qquad \Delta\Sigma_{N^*} = -0.02 + 0.01 \simeq 0.0$$

- Result $\Longrightarrow$ subtle cancellation between quark and diquark spin states
    - e.g. axial–vector diquark now has greater probability to have spin opposite nucleon

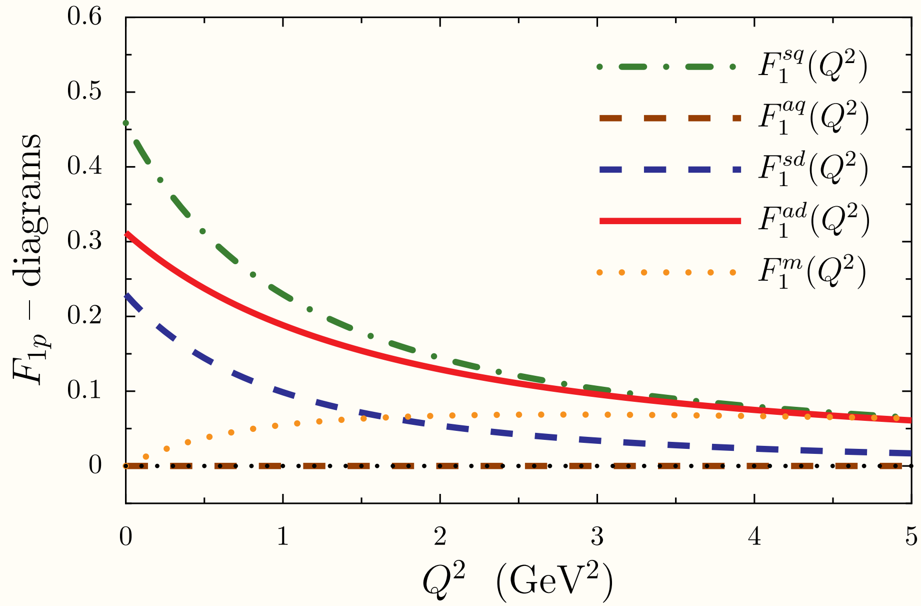
# *Nucleon and $N^*$ Form Factors*

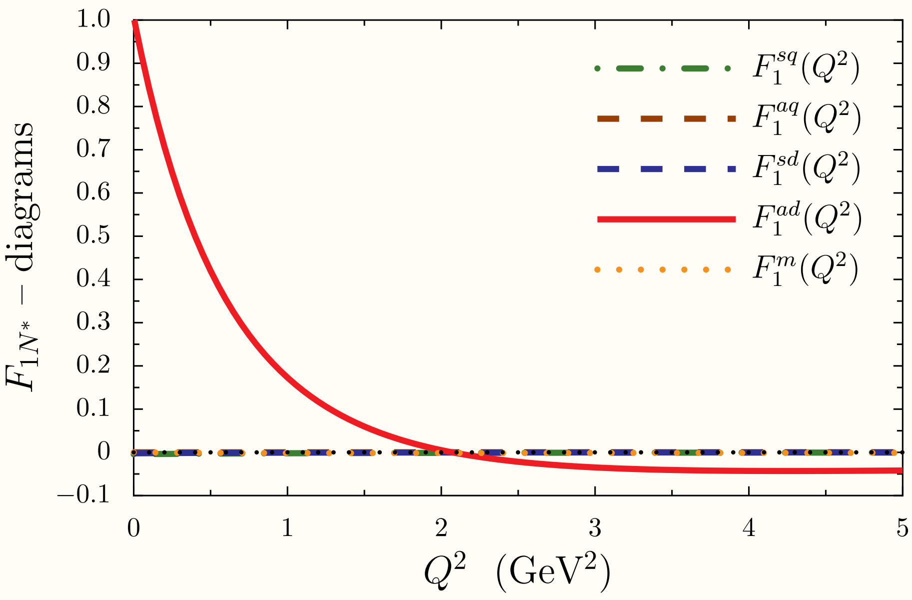


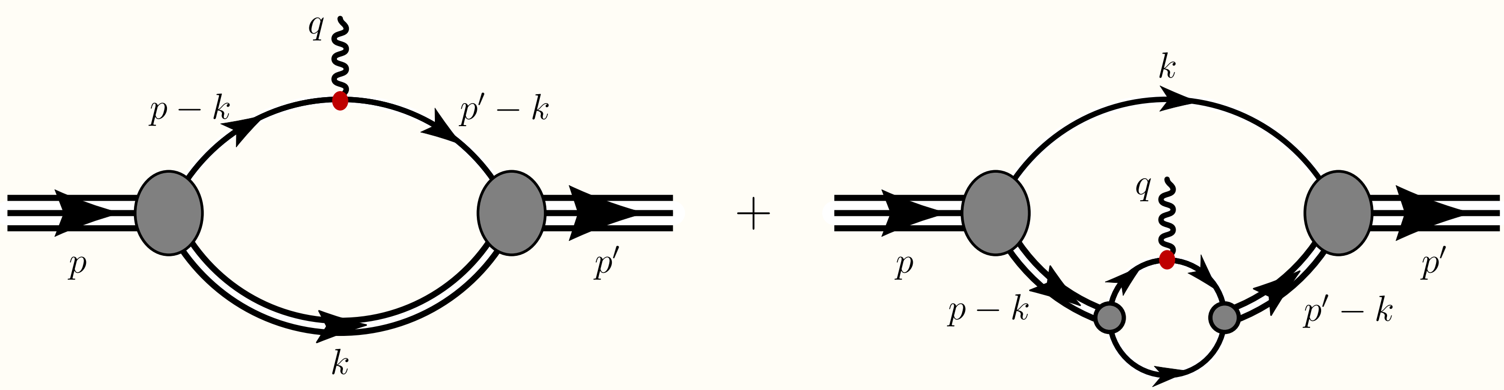

- Note these results are obtained within the constant mass function framework
  - therefore moderate to large $Q^2$ behaviour is poor
- Pion cloud effects have been ignored
  - expect magnetic moments and radii to be too small
- However we find $N^*$ radii are $10\%$ larger than the nucleons
- Find a zero in both $F_1$ and $F_2$ for Roper

# Nucleon and $N^*$ – $F_1$ Form Factors Results



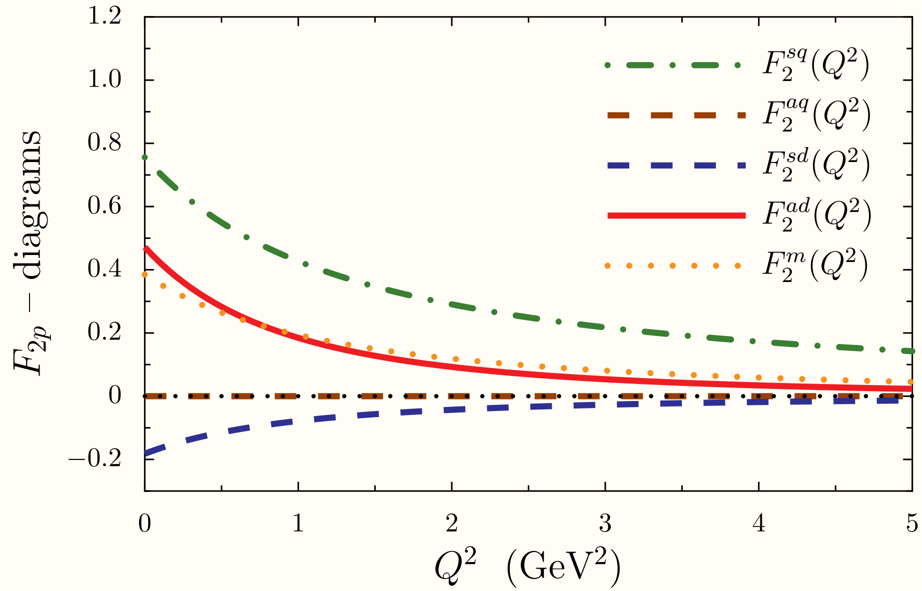



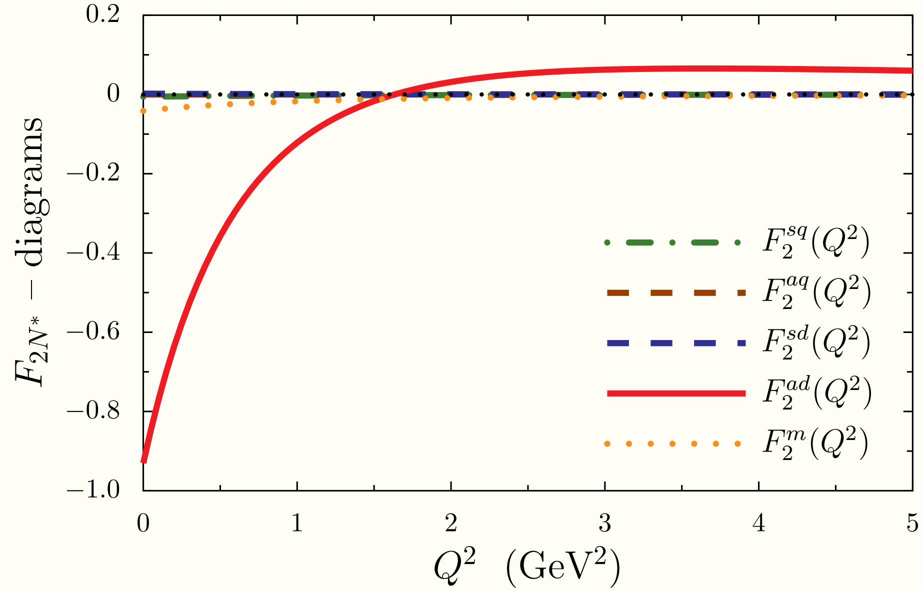

- Contributions originate from the following diagrams




- Find that $N^*$ form factors are axial–vector diquark dominated

# Nucleon and $N^*$ – $F_2$ Form Factors Results

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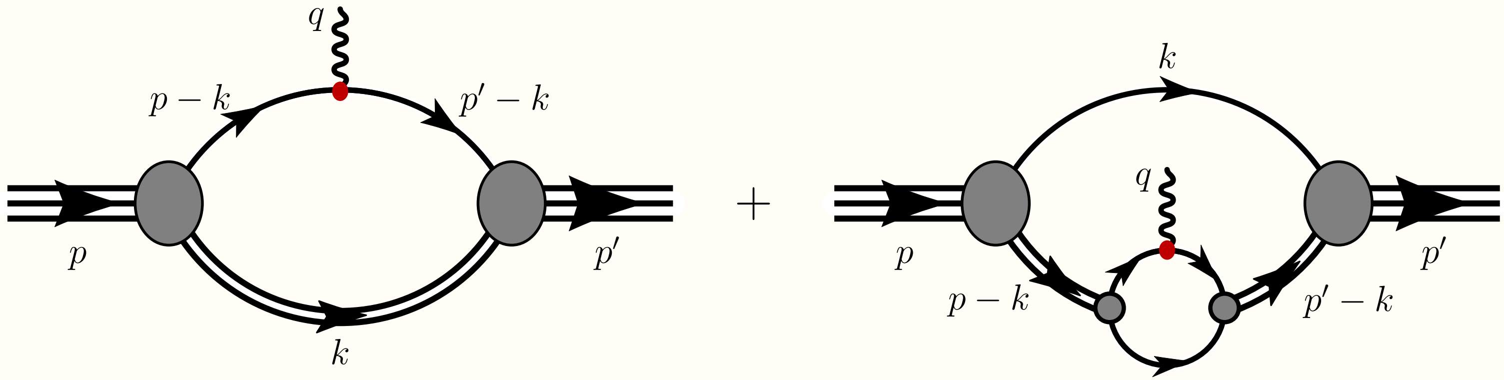
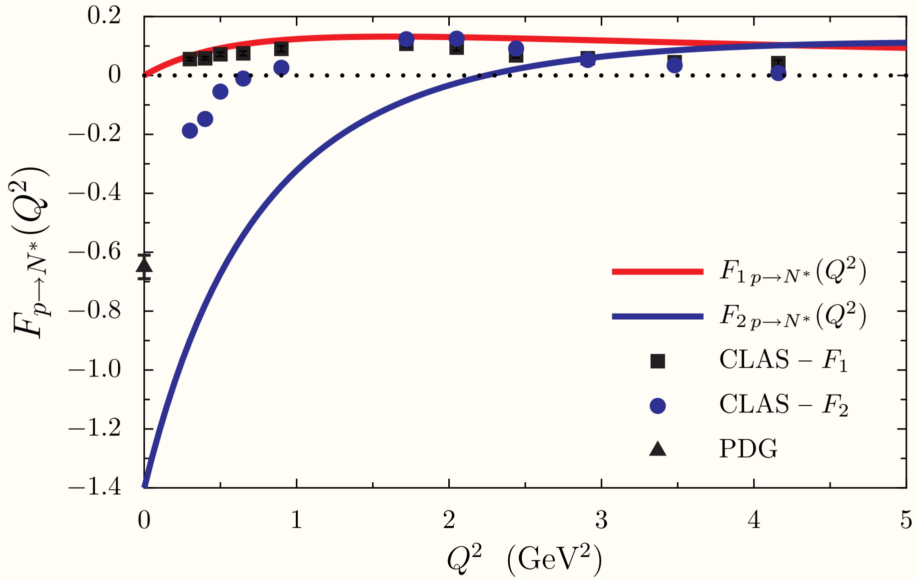
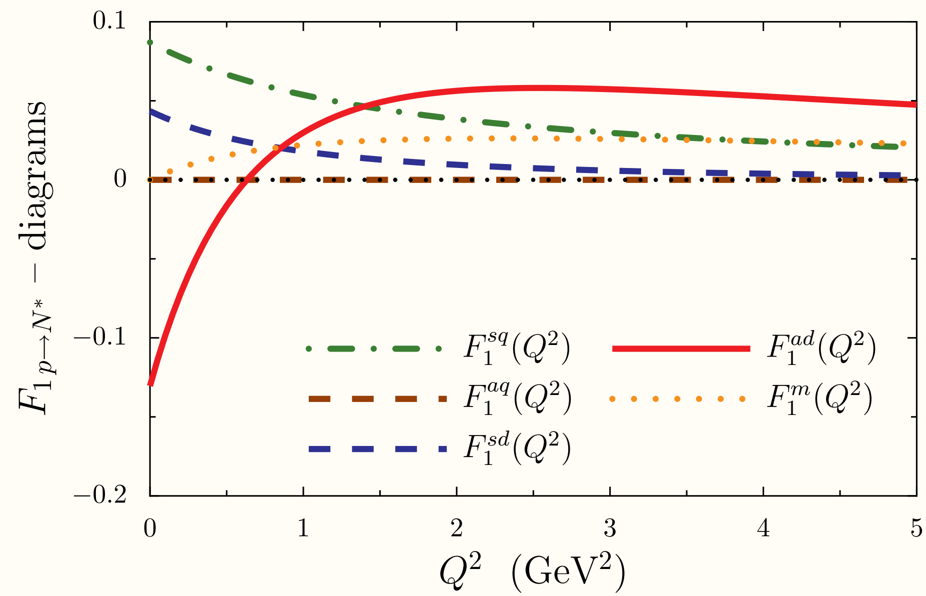
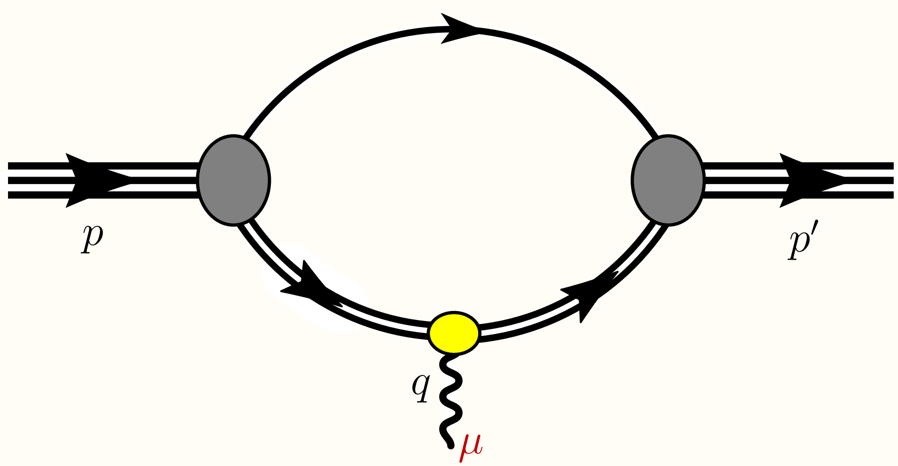


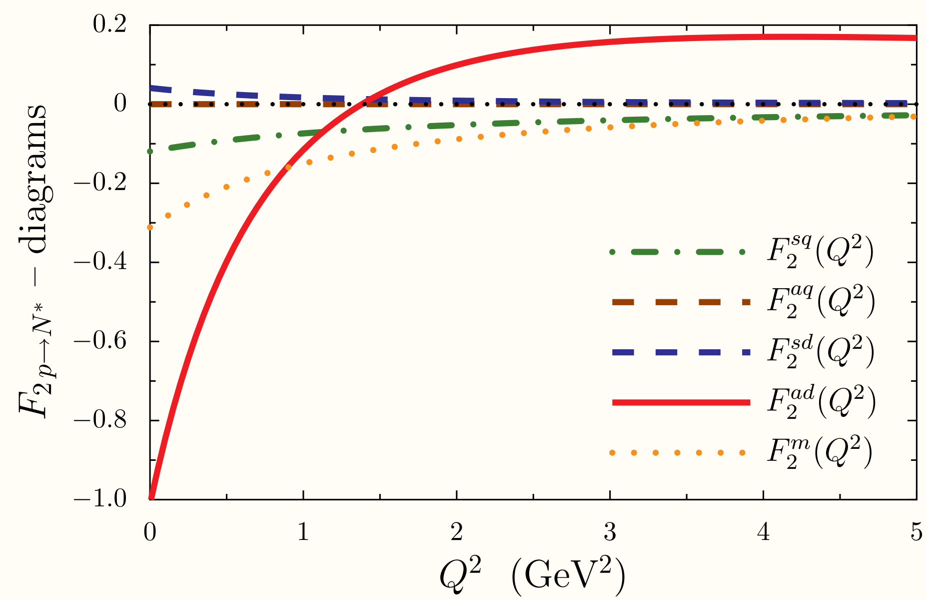

- Find that $N^*$ form factors are axial–vector diquark dominated

# $N \to N^*$ Transition Form Factors Results

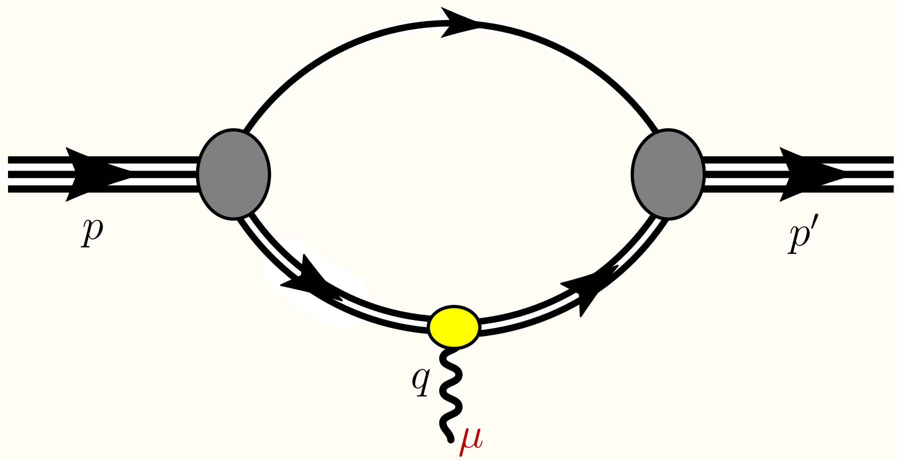


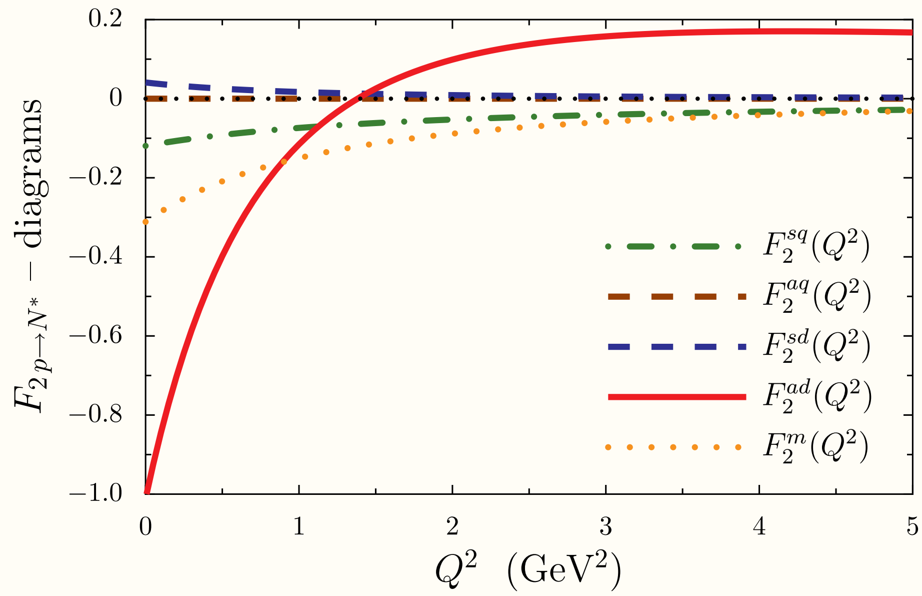



# Why the Zero




- The photon–axial-vector diquark vertex has the form

$$\Lambda^{\mu,\alpha\beta}_{ax} = \left[ g^{\alpha\beta}\, F_1(Q^2) - \frac{q^\alpha q^\beta}{2M_a^2}\, F_2(Q^2) \right] (p+p')^\mu - \left( q^\alpha\, g^{\mu\beta} - q^\beta\, g^{\mu\alpha} \right)\, F_3(Q^2)$$

- The three axial-vector diquark form factors are positive definite
- Cancellations between pieces of diagram give zero in $F_{2\,p\to N^*}$
- This zero is directly related to the zeros in the $N^*$ form factors

# *Conclusion*

- A thorough understanding of hadron structure requires a nonperturbative, symmetry preserving framework
  - Poincaré covariance, chiral symmetry, current conservation, etc
- Dyson–Schwinger equations provides such a framework
  - single approach that combines UV and IR physics
  - incorporates both quarks AND gluons
- Confronting experiment within the DSE framework will hopefully shed light on the non–perturbative structure of QCD
- Tried to highlight that form factors possibly provide the best empirical constraints on non–perturbative structure within the DSE framework
  - In particular the dressed quark–gluon vertex
- We have outlined a simple but intuitive picture regarding $N \to N^*$ transition form factors $\Longleftrightarrow$ axial–vector diquark dominance
  - however much work still remains before a robust picture emerges